The Affects of Market Fluctuation and Social Networks on the Collapse of Ponzi Schemes

Randy M. Casstevens and Jennifer S. Trueblood
Graduate Workshop, Santa Fe Institute

Introduction

In this paper, we investigate the role of market instability and social connections on participation in Ponzi schemes. Before making investment decisions, individuals examine the market by evaluating the possible return on investment. However, individuals do not make decisions in isolation. Rather, investment decisions are often influenced by the decisions of others. We establish that under these assumptions, the Ponzi scheme will eventually collapse.

The Computation Model

Social Network

In our model, we define a social network with 100 vertices and situate potential investor agents on this graph. In order to capture the structure of many real world business networks, we use a scale-free network generated by the BA model. The BA model captures real world structures by allowing for growth and preferential attachment. Our network grows by connecting new agents to at most five preexisting agents. In the this model, preferential attachment is incorporated by defining the probability $\Pi$ of a new vertex establishing an edge with existing vertex $i$ as

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$

where $k_i$ is the degree of vertex $i$.

Investment and Market Structures

If an agent chooses to invest in the scheme, they invest $1,000, and the returns for investing are adjusted according to the market. For simplicity, we assume that money invested in the Ponzi scheme is not reinvested. Therefore, investor agents have no other investment opportunities other than the Ponzi scheme. We examine our model using two different market structures. First, we model the market with a fixed rate of return. Then we model the market with data from the S & P 500 to give a more realistic view of how a market fluctuates over time.
Evolution

We allow the system to evolve until the Ponzi scheme collapses, i.e. when the amount of money invested in the scheme falls below 0. At the beginning of each generation, the agents decide to invest, continue investing, or withdraw from the Ponzi scheme. Agents choose to invest and withdraw from the scheme based on probabilistic functions defined in terms of the return on investment (ROI) and social influence (SI). We define social influence as the percentage of neighbors participating in the scheme. Similar to Granovetter’s threshold model, we define a ROI threshold $\alpha (\phi)$ for investment (withdrawal) distributed to a normal distribution. Likewise, we define a SI threshold $\beta (\psi)$ for investment (withdrawal) distributed to a normal distribution. For an agent not participating in the scheme, we define the probability $\pi$ of investment as

$$
\pi = \begin{cases} 
    p, & \text{if } ROI \geq \alpha, SI < \beta \\
    q, & \text{if } ROI < \alpha, SI \geq \beta \\
    p + q, & \text{if } ROI \geq \alpha, SI \geq \beta \\
    0, & \text{if } ROI < \alpha, SI < \beta 
\end{cases}
$$

where $p, q$ are selected from a normal distribution. For an agent already invested in the scheme, we define the probability $\pi'$ of withdrawal as

$$
\pi' = \begin{cases} 
    p', & \text{if } ROI < \phi, SI \geq \psi \\
    q', & \text{if } ROI \geq \phi, SI < \psi \\
    p' + q', & \text{if } ROI < \phi, SI < \psi \\
    0, & \text{if } ROI \geq \phi, SI \geq \psi 
\end{cases}
$$

where $p', q'$ are selected from a normal distribution.

Results

Results from a fixed rate of return

For our initial test, we model the market with a fixed rate of return of 7%. The probabilities $p, q, p', q'$ are selected from a normal distribution with mean $.05$ and standard deviation $.01$. We allow the probability of investment to be relatively small to account for risk aversion. The thresholds are defined as the following:

$$
\alpha \sim N(.07, .01) \\
\beta \sim N(.5, .1) \\
\phi = \alpha \\
\psi = \beta
$$

We find that as time progresses the scheme slowly loses money until it goes broke (see Figure 1).
Figure 1. Amount Invested in the Ponzi scheme

Results from S & P 500 data

Next, we test the model by using weekly S & P 500 data from January 1993 to June 2009. We use the same thresholds and probability distributions as the initial test. We found that high returns initially attract investors, but as the returns drop, the number of investors decrease ultimately leading the the collapse of the system. Figure 2 illustrates the collapse, and Figure 3 shows the relationship between the actual returns (blue) and the number of investors (red).

Figure 2. Amount Invested in the Ponzi scheme with S & P 500 data

Future Directions

Once investors begin to withdraw from the scheme, there is an increased possibility of exposure. In a real Ponzi scheme, measures would be taken to avoid losings investors. This may involve greater inflation of returns during economic downturns. Such an addition to the current model might allow one to capture the dynamics of the Bernie Madoff scandal. Also, the growth of the Ponzi scheme could be restricted in order to delay collapse. In the Madoff scheme, the number of participants was capped thereby creating a cue of potential
investors. As current investors withdrew from the scheme, they were replaced with investors in the cue. This strategy allowed Madoff to perpetuate the scam until the economic crisis in 2008.

![Figure 3. Number of Investors with S & P 500 data](image)

*Figure 3. Number of Investors with S & P 500 data*